

**OPTIMAL NUMERICAL SOLUTION OF A SECOND DERIVATIVE HYBRID
BLOCK BACKWARD DIFFERENTIATION FORMULA FOR FORECASTING
AGRICULTURAL PRODUCTION IN ADAMAWA STATE, NIGERIA**

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ABSTRACT

This study presents optimal numerical solutions based on a second-derivative hybrid block backward differentiation formula to forecast agricultural production in Adamawa State, Nigeria. Agricultural production forecasting refers to the process of predicting future crop yields, livestock output, or overall agricultural productivity in the region using various tools and methods. These predictions are based on advanced numerical methods. Moreover, using advanced numerical method in agricultural production brings several significant benefits, particularly in improving decision-making, efficiency and sustainability. These methods are especially valuable when applied to forecasting, optimization and modelling complex agricultural systems. The research was carried out with the aim of developing and applying an advanced numerical method for forecasting agricultural production in Adamawa State. By applying computational methods and algorithms, farmers and agricultural experts can optimize crop yields and enhance overall efficiency in the agricultural sector. Technologies such as precision farming, weather forecasting, and data analysis can help make better-informed decisions, leading to increased productivity. In addition, the hybrid block backward differentiation formula (BDF) is applied to agricultural production forecasting by formulating the production process as a system of differential equation. The method provides reliable numerical approximations, enabling accurate prediction of future agricultural output in Adamawa State. The order and error constant have been investigated. The convergence analysis reveals that the methods $k = 4$ and their block forms are A-stable. The newly constructed method are applied to Vander pol stiff ordinary differential equations arising from real life situation has shown that it is efficient and compare favourably with other existing methods and the exact solutions. The method is also efficient because from the graphical solutions, the approximate solution matched toward the exact solution. The policy implication of this research is that the application of

this method to predict agricultural production in Adamawa State would serve as reliable computational tool for forecasting crop yields and assessing the impact of environmental and economic variables on production outcomes.

Keywords: Agriciulture production, backward Differentiation, Block Method, Hybrid, Second Derivate

1. Introduction

Adamawa's main food and cash crops include maize, rice, yam, cassava, sorghum, millet, groundnut, cotton, sesame, cowpea and others. Agriculture is the major livelihood for a large share of the population. Maize (*Zea mays* L.), a member of the grass family (Gramineae), originated in South and Central America and was introduced to West Africa by the Portuguese in the 10th century. It is one of the most important cereal crops in Nigeria, Valued not only for the large number of farmers who cultivate it but also for its substantial economic important. (Oladejo et al., 2012). According to Jenkins and Scanlan (2001), food constitutes a fundamental human necessity and occupies a central position in discourses on human rights and social development. . Scientific methods, as Tripp (2004) notes, can significantly improve the quality of farms, inputs, products, and farming systems. For this reason, both developed and developing nations are making concerted efforts to increase their food production capacity. The goal is to ensure long-term food availability and to promote balanced diets that positively influence national economic development (Macnamara, 1973). Many countries rely heavily on agriculture to drive their economies, as it generates employment, stimulates economic growth, increases marketable surplus, boosts tax revenue, and supplies essential raw materials and food (Obianefo et al., 2020). Through food consumption, individuals are connected to a global network of farmers, retailers, producers, and distributors. This vast system is responsible for feeding the world's 7.9 billion people with safe and nutritious food. Since agriculture occupies about 40% of the Earth's land area, it plays a vital role in ensuring environmental and economic sustainability. Many of these challenges can be modeled using differential equations, and numerical methods have proven to be effective tools for solving such complex problems (Famurewa and Olorunsola, 2014). To justify the effectiveness of the newly proposed numerical method, this research will consider both linear and nonlinear systems of first-order differential equations. The numerical results obtained will be compared with those from existing methods in the literature to evaluate performance and accuracy. Developing mathematical models is a crucial step toward achieving efficient water use. Accurate estimation of temporal and spatial variations in

soil moisture, evaporation, and transpiration is necessary for sustainable water resource management, especially in arid and semi-arid regions (Aggarwal, 1995; Addiscott et al., 1995; Scanlon et al., 2002; Garatuza-Payan et al., 1998). Gustafson (1958) proposed various solution methods for such models, including iterating the value function, iterating the marginal value function, and approximating the storage rule using a piecewise linear function, with his work focusing primarily on value function iteration. The originality and distinctiveness of this research lie in the development of an efficient numerical scheme for accurately predicting agricultural production in Adamawa State.

2. Material and Method

Adamawa is located in northeastern Nigeria with yola as its capital, covers about 36,917 square kilometers and share an international boundary with the republic of Cameroon. This is characterized by diverse topography and a tropical climate with distinct wet dry seasons, supporting extensive Agricultural activities. The present hybrid block method was constructed using the numerical formulations and higher-derivative approximations presented by Famurewa (2017) as foundational material. The approach provided the basis for determining the continuous coefficients and stability properties adopted in this study for predicting agricultural production in Adamawa State.

2.1 Methodology

A general form of the continuous of the k – step linear multistep method as defined by Sunday (2022) is

$$y(x) = \sum_{j=0}^{t-1} \alpha_j(x) y_{n+j} + h \sum_{j=0}^{m-1} \beta_j(x) f_{n+j} \quad (1)$$

and its extension it to the second derivative gives the general form of the continuous k -step 2nd derivative linear multistep method as:

$$y(x) = \sum_{j=0}^{r-1} \alpha_j(x) y_{n+j} + h \sum_{j=0}^{t-1} \beta_j(x) f_{n+j} + h^2 \sum_{j=0}^{m-1} \gamma_j(x) \quad (2)$$

where h is the step length, s, t the number of distinct collocation points and r the number of interpolation points. Our aim is to utilize not only the interpolation point $\{x_i\}$ but also several collocation point on the interpolation interval and to fit $y(x)$ for $y'(x)$ and $y''(x)$. We impose the following conditions

$$y(x_{n+j}) = y_{n+j}, \quad (j = 0.1 \dots \dots r - 1) \quad (3)$$

$$y'(c_{n+j}) = \bar{y}'_{n+j}, \quad (j = 0.1 \dots t-1) \quad (4)$$

$$y''(c_{n+j}) = \bar{y}''_{n+j}, \quad (j = 0.1 \dots m-1) \quad (5)$$

Where $\{c_{n+j}\}$ are collocation points distributed on the step-points array, y_{n+j} is the interpolation data of $y(x)$ on x_{n+j} and $\bar{y}'_{n+j}, \bar{y}''_{n+j}$ are the collocation data of $y'(x)$ and $y''(x)$ respectively on $\{c_{n+j}\}$. Expressing equations (3), (4) and (5) in the matrix-vector form gives:

$$DC = I \quad (6)$$

Where I is the identity matrix of dimension $(t+m) \times (t+m)$. The matrix D is defined as

$$D = \begin{bmatrix} 1 & x_n & x_n^2 & \dots & x_n^{t+m-1} \\ 1 & x_{n+1} & x_{n+1}^2 & \dots & x_{n+1}^{t+m-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n+t-1} & x_{n+t-1}^2 & \dots & x_{n+t-1}^{t+m-1} \\ 0 & 1 & 2\bar{x}_0 & \dots & (t+m-1)\bar{x}_0^{t+m-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & 2\bar{x}_{m-1} & \dots & (t+m-1)\bar{x}_{m-1}^{t+m-2} \\ 0 & 0 & 1 & \dots & (t+m-2)\bar{x}_{m-1}^{t+m-3} \end{bmatrix} \quad (7)$$

$$C = \begin{bmatrix} \alpha_{0,1} & \alpha_{1,1} & \dots & \alpha_{t-1,1} & h\beta_{0,1} & \dots & h\beta_{m-1,1} \\ \alpha_{0,2} & \alpha_{1,2} & \dots & \alpha_{t-1,2} & h\beta_{0,2} & \dots & h\beta_{m-1,2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{0,t+m} & \alpha_{1,t+m} & \dots & \alpha_{t-1,t+m} & h\beta_{0,t+m} & \dots & h\beta_{m-1,t+m} \end{bmatrix} \quad (8)$$

The columns of $C = D^{-1}$ with aids of MAPLE 18 we will obtain the continuous coefficient $\alpha_j(x)$, $\beta_j(x)$ and $\gamma_j(x)$.

2.2 Derivation of the Four-Step New Hybrid Block Method

In this case, $k = 4$, $m = 6$ and $t = 4$ and its continuous form expressed as:

$$y(x) = \alpha_0(x)y_n + \alpha_1(x)y_{n+1} + \alpha_2(x)y_{n+2} + \alpha_3(x)y_{n+3} + h \left[\beta_3(x)f_{n+3} + \beta_{n+\frac{7}{2}}(x)f_{n+\frac{7}{2}} + \beta_4(x)f_{n+4} \right] + h^2 \left[\gamma_3(x)g_{n+3} + \gamma_{n+\frac{7}{2}}(x)g_{n+\frac{7}{2}} + \gamma_4(x)g_{n+4} \right] \quad (9)$$

The matrix D in (7) becomes

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 & 16 & 32 & 64 & 128 & 256 & 512 \\ 1 & 3 & 9 & 27 & 81 & 243 & 729 & 2187 & 6561 & 19683 \\ 0 & 1 & 6 & 27 & 108 & 405 & 1458 & 5103 & 17496 & 59049 \\ 0 & 1 & 7 & \frac{147}{4} & \frac{343}{2} & \frac{12005}{16} & \frac{50421}{16} & \frac{823543}{64} & \frac{823543}{16} & \frac{15883209}{256} \\ 0 & 1 & 16 & 48 & 256 & 1280 & 6144 & 28672 & 131072 & 589824 \\ 0 & 0 & 2 & 18 & 108 & 540 & 2430 & 10206 & 40824 & 157464 \\ 0 & 0 & 2 & 21 & 147 & \frac{1715}{2} & \frac{36015}{8} & \frac{352947}{16} & \frac{823543}{4} & \frac{7411887}{16} \\ 0 & 0 & 2 & 24 & 192 & 1280 & 7680 & 43008 & 229376 & 1179648 \end{bmatrix} \quad (10)$$

Using the Maple software, the inverse of the matrix in (10) is obtained and this yield the element C

$$\begin{aligned} \alpha_0(t) &= 1 - \frac{234125920t}{66203803h} + \frac{1058017366t^2}{198611409h^2} - \frac{8100002098t^3}{1787502681h^3} + \frac{7673435699t^4}{3177782544h^4} \\ &\quad - \frac{4002713845t^5}{4766673816h^5} + \frac{151898333t^6}{794445636h^6} - \frac{131343719t^7}{4766673816h^7} + \frac{7261225t^8}{3177782544h^8} \\ &\quad - \frac{297439t^9}{3575005362h^9} \\ \alpha_1(t) &= \frac{1482953346t}{66203803h} - \frac{7521914421t^2}{132407606h^2} + \frac{330985811793t^3}{529630424h^3} - \frac{41289147573t^4}{1059260848h^4} \\ &\quad + \frac{3995238645t^5}{264815212h^5} - \frac{1966246349t^6}{529630424h^6} + \frac{300533571t^7}{529630424h^7} - \frac{52193073t^8}{1059260848h^8} \\ &\quad + \frac{246493t^9}{132407606h^9} \\ \alpha_2(t) &= -\frac{16549650432t}{66203803h} + \frac{150233937670t^2}{66203803h^2} - \frac{62985422124t^3}{66203803h^3} + \frac{694752471327t^4}{1059260848h^4} \\ &\quad - \frac{145541484099t^5}{529630424h^5} + \frac{19097240321t^6}{264815212h^6} - \frac{6157374129t^7}{529630424h^7} \\ &\quad + \frac{1118337981t^8}{1059260848h^8} - \frac{5486995t^9}{132407606h^9} \end{aligned}$$

$$\begin{aligned}
 \alpha_3(t) &= \frac{15300823006t}{66203803h} - \frac{280953917489t^2}{397222818h^2} + \frac{12775989503735t^3}{14300021448h^3} \\
 &\quad - \frac{1968063406961t^4}{3177782544h^4} + \frac{620980887563t^5}{2383336908h^5} - \frac{108988499545t^6}{1588891272h^6} \\
 &\quad + \frac{528429087411t^7}{4766673816h^7} - \frac{3205695949t^8}{3177782544h^8} + \frac{141790993t^9}{3575005362h^9} \\
 \beta_3(t) &= -\frac{104925697828t}{66203803} + \frac{341043966557t^2}{66203803h} - \frac{16595819651435t^3}{2383336908h^2} \\
 &\quad - \frac{2739951759341t^4}{924829680245t^5} + \frac{173036515573t^6}{173036515573t^6} \\
 &\quad - \frac{529630424h^3}{89071107353t^7} + \frac{397222818h^4}{5712407281t^8} + \frac{264815212h^5}{266002309t^7} \\
 &\quad + \frac{794445636h^6}{529630424h^6} + \frac{595834227h^6}{595834227h^6} \\
 \beta_7(t) &= -\frac{83626242048t}{66203803} + \frac{268119161856t^2}{66203803h} - \frac{356698849536t^3}{66203803h^2} + \frac{260456702848t^4}{66203803h^3} \\
 &\quad - \frac{115091418624t^5}{66203803h^4} + \frac{31689019904t^6}{66203803h^5} - \frac{5330320384t^7}{66203803h^6} + \frac{502296192t^8}{66203803h^7} \\
 &\quad - \frac{20350208t^9}{66203803h^8} \\
 \beta_4(t) &= -\frac{35519373477t}{66203803} + \frac{116694845316t^2}{66203803h} - \frac{159906917844t^3}{66203803h^2} + \frac{966221942497t^4}{529630424h^3} \\
 &\quad - \frac{221634829401t^5}{31781664131t^6} + \frac{11169131695t^7}{11169131695t^7} \\
 &\quad - \frac{264815212h^4}{2204644275t^8} + \frac{132407606h^5}{11719133t^9} - \frac{264815212h^6}{264815212h^6} \\
 &\quad - \frac{2204644275t^8}{529630424h^6} - \frac{11719133t^9}{66203803h^7} \\
 \gamma_3(t) &= \frac{34888627788th}{66203803} - \frac{111500926831t^2}{66203803} + \frac{1774489229219t^3}{794445636h} - \frac{861681396303t^4}{529630424h^2} \\
 &\quad + \frac{95063600225t^5}{132407606h^3} - \frac{52356049989t^6}{264815212h^4} + \frac{8822796545t^7}{264815212h^5} - \frac{1668916303t^8}{529630424h^6} \\
 &\quad + \frac{25496797t^9}{198611409h^7} \\
 \gamma_7(t) &= \frac{43862853888th}{66203803} - \frac{142771187520t^2}{66203803} + \frac{193459195440t^3}{66203803h} - \frac{144269687048t^4}{66203803h^2} \\
 &\quad + \frac{65262532496t^5}{66203803h^3} - \frac{18434627648t^6}{66203803h^4} + \frac{3187245248t^7}{66203803h^5} - \frac{309244280t^8}{66203803h^6} \\
 &\quad + \frac{12919424t^9}{66203803h^7} \\
 \gamma_4(t) &= \frac{4744886580th}{66203803} - \frac{1262580909t^2}{132407606} + \frac{21491110854t^3}{66203803h} - \frac{130363908235t^4}{529630424h^2} \\
 &\quad + \frac{30035908411t^5}{264815212h^3} - \frac{2164257200t^6}{66203803h^4} + \frac{1529640817t^7}{264815212h^5} - \frac{303795841t^8}{529630424h^6} \\
 &\quad + \frac{1625923t^9}{66203803h^7}
 \end{aligned}$$

Evaluating (9) at the following points at $t_n, t_{n+1}, t_{n+2}, t_{n+3}, t_{n+\frac{7}{2}}$ and t_{n+4} yields the following discrete methods which constitute the new four-step hybrid block method.

$$y_n - \frac{3223677609}{302290676} y_{n+1} + \frac{10764415215}{75572669} y_{n+2} - \frac{40136273927}{302290676} y_{n+3} \\ = \frac{3h}{151145338} [48720566651 f_{n+3} - 38302737408 f_{n+\frac{7}{2}} \\ - 16670692188 f_{n+4}] + \frac{3h^2}{2116034732} [66203803 g_n \\ + 223001853662 g_{n+3} + 285542375040 g_{n+\frac{7}{2}} + 31262580909 g_{n+4}]$$

$$y_n - \frac{410315328}{27650303} y_{n+1} - \frac{6737129775}{27650303} y_{n+2} + \frac{6299164144}{27650303} y_{n+3} \\ = -\frac{6h}{27650303} [6567229408 f_{n+3} - 5368439808 f_{n+\frac{7}{2}} \\ - 2171047635 f_{n+4}] + \frac{6h^2}{138251515} [66203803 g_{n+1} - 11255166555 g_{n+3} \\ - 13665192768 g_{n+\frac{7}{2}} - 1441772065 g_{n+4}]$$

$$y_n - \frac{165064851}{3285214} y_{n+1} - \frac{1572484455}{1642607} y_{n+2} + \frac{3306748547}{3285214} y_{n+3} \\ = -\frac{3h}{1642607} [2923741511 f_{n+3} - 2607879168 f_{n+\frac{7}{2}} - 893402838 f_{n+4}] \\ - \frac{3h^2}{1642607} [66203803 g_{n+2} - 1127687181 g_{n+3} - 1201554368 g_{n+\frac{7}{2}} \\ - 116535519 g_{n+4}]$$

$$y_{n+\frac{7}{2}} = -\frac{252125}{271170777088} y_n + \frac{8177463}{271170777088} y_{n+1} - \frac{330351875}{271170777088} y_{n+2} \\ + \frac{271493203625}{271170777088} y_{n+3} \\ + \frac{105h}{135585388544} [305650975 f_{n+3} + 327084032 f_{n+\frac{7}{2}} + 11410875 f_{n+4}] \\ + \frac{11025h^2}{135585388544} [244265 g_{n+3} - 353728 g_{n+\frac{7}{2}} - 12705 g_{n+4}]$$

$$y_{n+4} = -\frac{181}{66203803} y_n + \frac{5608}{66203803} y_{n+1} - \frac{204336}{66203803} y_{n+2} + \frac{66402712}{66203803} y_{n+3} \\ + \frac{12h}{66203803} [1624412 f_{n+5} + 2764800 f_{n+\frac{7}{2}} + 1111633 f_{n+4}] \\ + \frac{72h^2}{66203803} [29086 g_{n+3} + 27264 g_{n+\frac{7}{2}} - 10999 g_{n+4}]$$

3. Analysis of the Four-Step New Hybrid Block Method

In this section, the analysis of the proposed four-step hybrid block method is presented. Their convergence is examined, while their order and region of absolute stability are determined and presented graphically.

With the help of MAPLE 18, the method exhibits a uniform order of the newly constructed hybrid block method $(9,9,9,9,9)^T$ and error constants

$$\left(-\frac{29705780197}{13542622284800}, \frac{9123339877}{3716200723200}, \frac{2560044691}{441532761600}, \frac{2065595}{208259156803584}, \frac{36541}{1112223890400}\right)^T$$

The region of absolute stability for the new method is obtained by writing the method in the form of stability

$$D(z) = (r(A - Cz - D1z^2) - B) \quad (11)$$

and applying the test problem

$$y' = \lambda y, \lambda < 0 \quad \text{to yield}$$

$$Y_{w+1} = D(z)Y_w, z = \lambda h$$

The matrix $D(z)$ has eigenvalues $\{\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_k\} = \{0, 0, 0, \dots, \lambda_k\}$

Where the dominant eigenvalue λ_k is the stability function $R(z): \mathbb{C} \rightarrow \mathbb{C}$ which is the rational function with real coefficients.

$$\begin{aligned}
 D(z) = & \frac{148723786519379802617386500}{3432403408005612699149} r^5 - \frac{79256727559978083849157200}{3432403408005612699149} z r^4 \\
 & + \frac{154163799540377831591446365}{192214590848314311152344} z^2 r^4 \\
 & + \frac{26751220989211377462385089}{10116557413069174271176} z^4 r^7 \\
 & + \frac{154396448085235986234825471}{384429181696628622304688} z^3 r^4 \\
 & - \frac{32382163038086470744436997}{384429181696628622304688} z^4 r^4 \\
 & - \frac{7365213332193285670696587}{192214590848314311152344} z^6 r^4 - \frac{2305830121534676848666311}{384429181696628622304688} z^7 r^4 \\
 & - \frac{194335311335679629936955}{384429181696628622304688} z^8 r^4 - \frac{148723786519379802617386500}{3432403408005612699149} r^4 \\
 & - \frac{12926992348055111496890517}{13729613632022450796596} z^9 r^5 \\
 & + \frac{319388776824835488589560}{3432403408005612699149} z^{10} r^5 \\
 & - \frac{515638418517541126620388800}{3432403408005612699149} z r^5 \\
 & + \frac{6891916741864186293458366085}{27459227264044901593192} z^2 r^5 \\
 & - \frac{7366882296890413460975105385}{27459227264044901593192} z^3 r^5 \\
 & + \frac{11268681176988580136000187375}{54918454528089803186384} z^4 r^5 \\
 & - \frac{6520666653248397090526218105}{54918454528089803186384} z^5 r^5 \\
 & + \frac{1466809452789187224924214227}{27459227264044901593192} z^6 r^5 \\
 & - \frac{1031521932880976661893610975}{54918454528089803186384} z^7 r^5 \\
 & + \frac{276875147409615663488983377}{54918454528089803186384} z^8 r^5
 \end{aligned}$$

$$\begin{aligned} \bar{D}(z) = & -\frac{79256727559978083849157200}{3432403408005612699149}r^4 + \frac{154163799540377831591446365}{96107295424157155576172}zr^4 \\ & + \frac{80253662967634132387155267}{10116557413069174271176}z^2r^4 \\ & + \frac{154396448085235986234825471}{96107295424157155576172}z^3r^4 \\ & - \frac{161910815190432353722184985}{384429181696628622304688}z^4r^4 \\ & - \frac{22095639996579857012089761}{96107295424157155576172}z^5r^4 \\ & - \frac{2305830121534676848666311}{54918454528089803186384}z^6r^4 - \frac{194335311335679629936955}{48053647712078577788086}z^7r^4 \\ & - \frac{148723786519379802617386500}{3432403408005612699149}r^4 \\ & - \frac{116342931132496003472014653}{13729613632022450796596}z^8r^5 \\ & + \frac{3193887768248354885895600}{3432403408005612699149}z^9r^5 - \frac{515638418517541126620388800}{3432403408005612699149}r^5 \\ & + \frac{6891916741864186293458366085}{13729613632022450796596}zr^5 \\ & - \frac{22100646890671240382925316155}{27459227264044901593192}z^2r^5 \\ & + \frac{11268681176988580136000187375}{13729613632022450796596}z^3r^5 \\ & - \frac{32603333266241985452631090525}{54918454528089803186384}z^4r^5 \\ & + \frac{4400428358367561674772642681}{13729613632022450796596}z^5r^5 \\ & - \frac{7220653530166836633255276825}{54918454528089803186384}z^6r^5 \\ & + \frac{276875147409615663488983377}{6864806816011225398298}z^7r^5 \end{aligned}$$

And substituting $D(z)$ and $\bar{D}(z)$ into a MATLAB code for plotting stability regions yields the absolute stability region of the newly developed hybrid block method.

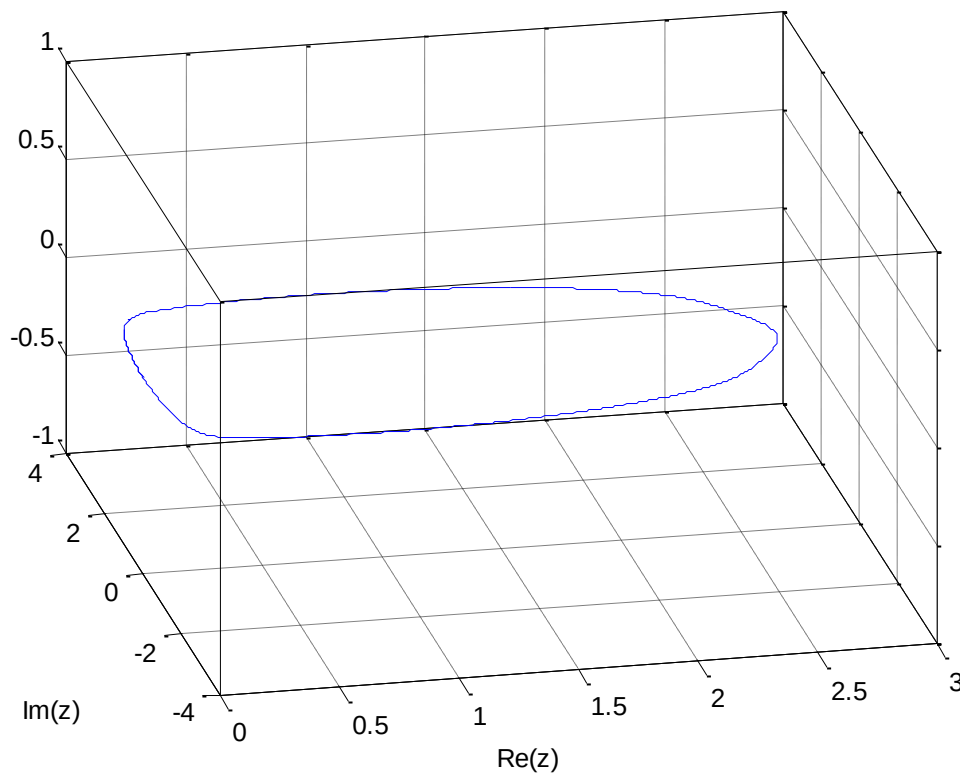


Figure 1

The region of absolute stability for newly developed method in figure 1. The shows that the method possesses a large stability region. Indicating its suitability for solving stiff initial value problem. The method is *A – stable* its region of absolute stability contains the entire left hand of the complex plane.

3.1 Convergence

The convergence of the new block method is established using the approach proposed by Fatunla (1991) Chollom et al. (2007) for linear multistep methods. Where the block methods are expressed as single *r – point* multistep method of the form

$$A^{(1)}Y_{w+1} = B^{(0)}Y_w + hC^{(1)}F_w + h^2D^{(1)}G_w \quad (12)$$

Where

$$y_{w+1} = (y_{n+1}, y_{n+2}, y_{n+3}, \dots, y_{n-k-1}, y_{n+k})^T$$

$$y_w = (y_{n-k+1}, y_{n-k+2}, y_{n-k+3}, \dots, y_{n-1}, y_n)^T$$

$$F_w = (f_{n+1}, f_{n+2}, y_{n+3}, \dots, f_{n-k-1}, f_{n+k})^T$$

$$G_w = (g_{n+1}, g_{n+2}, g_{n+3}, \dots, g_{n-k-1}, g_{n+k})^T$$

For $w = 0, \dots$ and $n = 0, k, \dots N - k$. The block method expressed in the form equation (12) yields

$$\begin{pmatrix} -\frac{3223677609}{302290676} & \frac{10764415215}{75572669} & -\frac{40136273927}{302290676} & 0 & 0 \\ \frac{410315328}{27650303} & -\frac{6737129775}{27650303} & \frac{6299164144}{27650303} & 0 & 0 \\ -\frac{165064851}{3285214} & -\frac{1572484455}{1642607} & \frac{3306748547}{3285214} & 0 & 0 \\ -\frac{8177463}{271170777088} & \frac{330351875}{271170777088} & \frac{271493203625}{271170777088} & 1 & 0 \\ -\frac{5608}{66203803} & \frac{204336}{66203803} & \frac{66402712}{66203803} & 0 & 1 \end{pmatrix} \begin{pmatrix} y_{n+1} \\ y_{n+2} \\ y_{n+3} \\ y_{n+\frac{7}{2}} \\ y_{n+4} \end{pmatrix} = \\
 \begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -\frac{252125}{271170777088} \\ 0 & 0 & 0 & 0 & -\frac{181}{66203803} \end{pmatrix} \begin{pmatrix} y_{n-4} \\ y_{n-3} \\ y_{n-2} \\ y_{n-1} \\ y_n \end{pmatrix} + \\
 h \left[\begin{pmatrix} 0 & 0 & \frac{46161699953}{151145338} & -\frac{57454106112}{75572669} & -\frac{25006038282}{75572669} \\ 0 & 0 & \frac{39403376448}{27650303} & \frac{32210638848}{27650303} & \frac{13026285810}{27650303} \\ 0 & 0 & -\frac{8771224533}{1642607} & \frac{7823637504}{1642607} & \frac{141063606}{86453} \\ 0 & 0 & \frac{32093352375}{135585388544} & \frac{16769445}{66203803} & \frac{1198141875}{135585388544} \\ 0 & 0 & \frac{19492944}{66203803} & \frac{33177600}{66203803} & \frac{13339596}{66203803} \end{pmatrix} \begin{pmatrix} f_{n+1} \\ f_{n+2} \\ f_{n+3} \\ f_{n+\frac{7}{2}} \\ f_{n+4} \end{pmatrix} + \right. \\
 \left. \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} f_{n-4} \\ f_{n-3} \\ f_{n-2} \\ f_{n-1} \\ f_n \end{pmatrix} \right]$$

$$h^2 \begin{bmatrix} 0 & 0 & \frac{47786111499}{151145338} & \frac{214156781280}{529008683} & \frac{13398248961}{302290676} \\ \frac{397222818}{138251515} & 0 & -\frac{13506199866}{27650303} & -\frac{81991156608}{138251515} & -\frac{1730126478}{27650303} \\ 0 & -\frac{198611409}{1642607} & -\frac{3383061543}{1642607} & \frac{3604663104}{1642607} & -\frac{349606557}{1642607} \\ 0 & 0 & \frac{2693021625}{135585388544} & -\frac{60935175}{2118521696} & -\frac{140072625}{135585388544} \\ 0 & 0 & \frac{2094192}{66203803} & \frac{1963008}{66203803} & -\frac{791928}{66203803} \end{bmatrix} \begin{pmatrix} g_{n+1} \\ g_{n+2} \\ g_{n+3} \\ g_{n+\frac{7}{2}} \\ g_{n+4} \end{pmatrix} +$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{198611409}{2116034732} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} g_{n-4} \\ g_{n-3} \\ g_{n-2} \\ g_{n-1} \\ g_n \end{pmatrix}$$

According to Fatunla (1991), a block method is said to be zero stable if $\lambda_{ij} = 1, 2, 3, \dots, k$ specified as $\rho(\lambda) = |\sum_{i=0}^k A^{(i)} \lambda^{k-i}| = 0$ (13)

Satisfies $\rho(\lambda) = |\lambda_j| \leq 1$, the multiplicity must not exceed two.

Where $A^{(0)} =$

$$\begin{pmatrix} -\frac{3223677609}{302290676} & \frac{10764415215}{75572669} & -\frac{40136273927}{302290676} & 0 & 0 \\ \frac{410315328}{27650303} & -\frac{6737129775}{27650303} & \frac{6299164144}{27650303} & 0 & 0 \\ -\frac{165064851}{3285214} & -\frac{1572484455}{1642607} & \frac{3306748547}{3285214} & 0 & 0 \\ -\frac{8177463}{271170777088} & \frac{330351875}{271170777088} & \frac{271493203625}{271170777088} & 1 & 0 \\ -\frac{5608}{66203803} & \frac{204336}{66203803} & \frac{66402712}{66203803} & 0 & 1 \end{pmatrix}$$

$$\begin{aligned}
 , A^{(1)} &= \begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -\frac{252125}{271170777088} \\ 0 & 0 & 0 & 0 & -\frac{181}{66203803} \end{pmatrix} B^{(0)} \\
 &= \begin{pmatrix} 0 & 0 & \frac{46161699953}{151145338} & -\frac{57454106112}{75572669} & -\frac{25006038282}{75572669} \\ 0 & 0 & \frac{39403376448}{27650303} & \frac{32210638848}{27650303} & \frac{13026285810}{27650303} \\ 0 & 0 & -\frac{8771224533}{1642607} & \frac{7823637504}{1642607} & \frac{141063606}{86453} \\ 0 & 0 & \frac{32093352375}{135585388544} & \frac{16769445}{66203803} & \frac{1198141875}{135585388544} \\ 0 & 0 & \frac{19492944}{66203803} & \frac{33177600}{66203803} & \frac{13339596}{66203803} \end{pmatrix}, B^{(1)} \\
 &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, C^{(0)} \\
 &= \begin{pmatrix} 0 & 0 & \frac{47786111499}{151145338} & \frac{214156781280}{529008683} & \frac{13398248961}{302290676} \\ \frac{397222818}{138251515} & 0 & -\frac{13506199866}{27650303} & -\frac{81991156608}{138251515} & -\frac{1730126478}{27650303} \\ 0 & -\frac{198611409}{1642607} & -\frac{3383061543}{1642607} & \frac{3604663104}{1642607} & -\frac{349606557}{1642607} \\ 0 & 0 & \frac{2693021625}{135585388544} & -\frac{60935175}{2118521696} & -\frac{140072625}{135585388544} \\ 0 & 0 & \frac{2094192}{66203803} & \frac{1963008}{66203803} & -\frac{791928}{66203803} \end{pmatrix}, C^{(1)} \\
 &= + \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{198611409}{2116034732} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}
 \end{aligned}$$

Substituting $A^{(0)}$ and $A^{(1)}$ into (13) gives the characteristic polynomial of the block method $\rho(\lambda) = \det(\lambda A^{(0)} - A^{(1)})$

$$= \det \lambda \begin{pmatrix} -\frac{3223677609}{302290676} & \frac{10764415215}{75572669} & -\frac{40136273927}{302290676} & 0 & 0 \\ \frac{410315328}{27650303} & -\frac{6737129775}{27650303} & \frac{6299164144}{27650303} & 0 & 0 \\ -\frac{165064851}{3285214} & -\frac{1572484455}{1642607} & \frac{3306748547}{3285214} & 0 & 0 \\ -\frac{8177463}{271170777088} & \frac{330351875}{271170777088} & \frac{271493203625}{271170777088} & 1 & 0 \\ -\frac{5608}{66203803} & \frac{204336}{66203803} & \frac{66402712}{66203803} & 0 & 1 \end{pmatrix}$$

$$- \begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -\frac{252125}{271170777088} \\ 0 & 0 & 0 & 0 & -\frac{181}{66203803} \end{pmatrix}$$

$$= \det \begin{bmatrix} -\frac{3223677609}{302290676}\lambda & \frac{10764415215}{75572669}\lambda & -\frac{40136273927}{302290676}\lambda & 0 & \lambda \\ \frac{410315328}{27650303}\lambda & -\frac{6737129775}{27650303}\lambda & \frac{6299164144}{27650303}\lambda & 0 & \lambda \\ -\frac{165064851}{3285214}\lambda & -\frac{1572484455}{1642607}\lambda & \frac{3306748547}{3285214}\lambda & 0 & \lambda \\ -\frac{8177463}{271170777088}\lambda & \frac{330351875}{271170777088}\lambda & \frac{271493203625}{271170777088}\lambda & \lambda & \frac{252125}{271170777088}\lambda \\ -\frac{5608}{66203803}\lambda & \frac{204336}{66203803}\lambda & \frac{66402712}{66203803}\lambda & 0 & \frac{66203984}{66203803}\lambda \end{bmatrix}$$

$$= \lambda^4(\lambda - 1) = 0$$

Therefore $\lambda_1 = 1, \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0$. The block method is convergent by Henrici (1962).

4. Numerical Experiment

In this study, the concern is the numerical experiment for stiff and non-stiff problems using developed scheme are shown in the table below.

Problem 1

$$\begin{aligned}y_1' &= -2y_1 + y_2 + 2\sin x \\ y_2' &= 998y_1 - 999y_2(\cos x - \sin x)\end{aligned}$$

$$h = 0.1$$

The exact solutions are:

$$\begin{aligned}y_1(x) &= 2e^{-x} + \sin x & y_1(0) &= 2 \\ y_2(x) &= 2e^{-x} + \cos x & y_2(0) &= 3\end{aligned}$$

Problem 2

Consider the stiffly linear problem of the system

$$\begin{aligned}y_1' &= 998y_1 + 1998y_2 & y_1(0) &= 1 \\ y_2' &= -999y_1 - 1999y_2 & y_2(0) &= 0 \\ h &= 0.1 & x &\in [0,1]\end{aligned}$$

Exact solution

$$\begin{aligned}y_1(x) &= 2e^{-x} - e^{-1000x} \\ y_2(x) &= -e^{-x} - e^{1000x}\end{aligned}$$

Problem 3

Vander pol's problem

$$\begin{aligned}y_1' &= y_2 & y_1(0) &= 2 \\ y_2' &= -y_1 + \mu y_2(1 - y_1^2) & y_2(0) &= 0\end{aligned}$$

$$0 \leq x \leq 70 \text{ and } h = 0.01\mu = 10$$

Table 1: Shows the Absolute errors for problem 1 computed using the method with $k = 4$

	Block method	Block method
x	$y_1(x)$	$y_2(x)$
0.1	2.45619E-09	4.66294E-10
0.2	2.43175E-09	5.66214E-10
0.3	9.82477E-12	1.86517E-12
0.4	9.72701E-12	2.26485E-12
0.5	1.14622E-13	2.17604E-14
0.6	1.13482E-13	2.64233E-14
0.7	1.25266E-15	2.3781E-16
0.8	1.88308E-13	3.57492E-14
0.9	1.86434E-13	4.34097E-14
1.0	2.82462E-17	5.36238E-18

Table 2: Shows the Absolute errors for problem 1 computed using the method with $k = 4$

	Block method	Block method
x	$y_1(x)$	$y_2(x)$
0.1	2.22245E-07	4.99600E-05
0.2	2.20034E-07	4.99600E-05
0.3	8.88982E-09	1.99840E-05
0.4	8.80136E-08	1.99804E-07
0.5	1.03715E-09	2.33147E-08
0.6	1.02683E-10	2.33147E-08
0.7	1.13345E-10	2.54796E-07
0.8	1.70388E-11	3.83027E-09
0.9	1.68693E-11	3.83027E-10
1.0	2.55582E-11	4.99600E-11

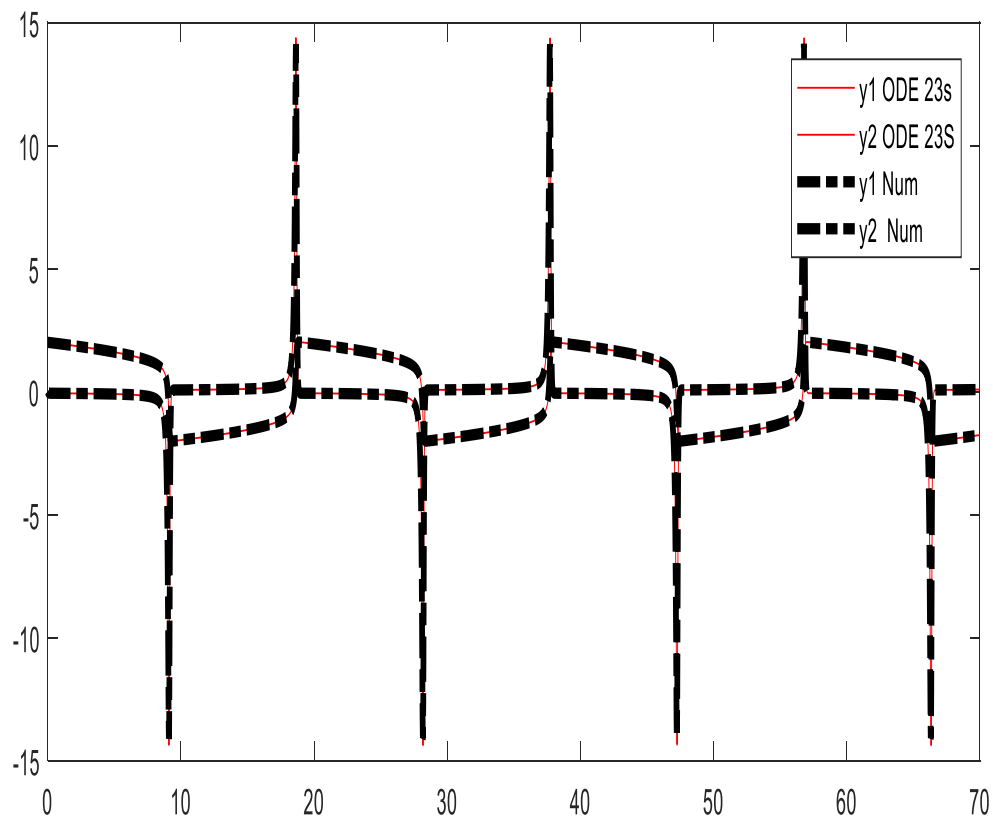


Figure 2

The graphical solution of Problem 3 obtained using the Method with $k = 4$ is presented in Figure 2. The figure clearly demonstrates the accuracy and stability of the proposed scheme in approximating the exact solution.

Table 3: Three different errors compared to developed method's errors

x	Error in Method $k = 4$	Error in Sunday 2018 Method	Error in <i>Umaru (2011)</i> Method $k = 5$	Error in Standard adams moulton method
0.1	2.45619E-09	1.7282e-12	7.9958000E-05	1.10E-02
0.2	2.43175E-09	6.3179e-12	9.9035000E-05	2.04E-02
0.3	9.82477E-12	1.4295e-11	1.0865900E-04	1.43E-02
0.4	9.72701E-12	2.5020e-11	1.2054400E-04	8.64E-03
0.5	1.14622E-13	3.8928e-11	1.3243700E-04	4.67E-03
0.6	1.13482E-13	5.5360e-11	2.7818900E-04	1.89E-03
0.7	1.25266E-15	7.4644e-11	3.2503600E-04	1.18E-03
0.8	1.88308E-13	9.6128e-11	3.5791200E-04	5.74E-04
0.9	1.86434E-13	1.2002e-10	3.9630900E-04	2.66E-04
1.0	2.82462E-17	1.4570e-10	4.3703900E-04	1.23E-04

The results shown in table 1-3 and figure 2 indicate that the newly developed hybrid block method provides a more accurate, stable, and efficient numerical tool for predicting agricultural production in Adamawa State. Its superior performance compared to existing methods (Sunday, 2018; Umaru, 2011) highlights its potential for broader application in agricultural modeling and forecasting. The method's ability to produce reliable predictions makes it a valuable analytical tool for policymakers, researchers and agricultural planners seeking to improve food production strategies and ensure sustainable agricultural development in the region.

5. Conclusion and Policy Implication

This study developed a hybrid block backward differentiation formula for predicting agricultural production in Adamawa State. The numerical experiments conducted on test problems demonstrated that the proposed method is accurate, efficient and stable for solving ordinary differential equations.

The policy implication of this research is that the application of this method to predict agricultural production in Adamawa State would serve as reliable computational tool for forecasting crop yields and assessing the impact of environmental and economic variables on production outcomes.

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